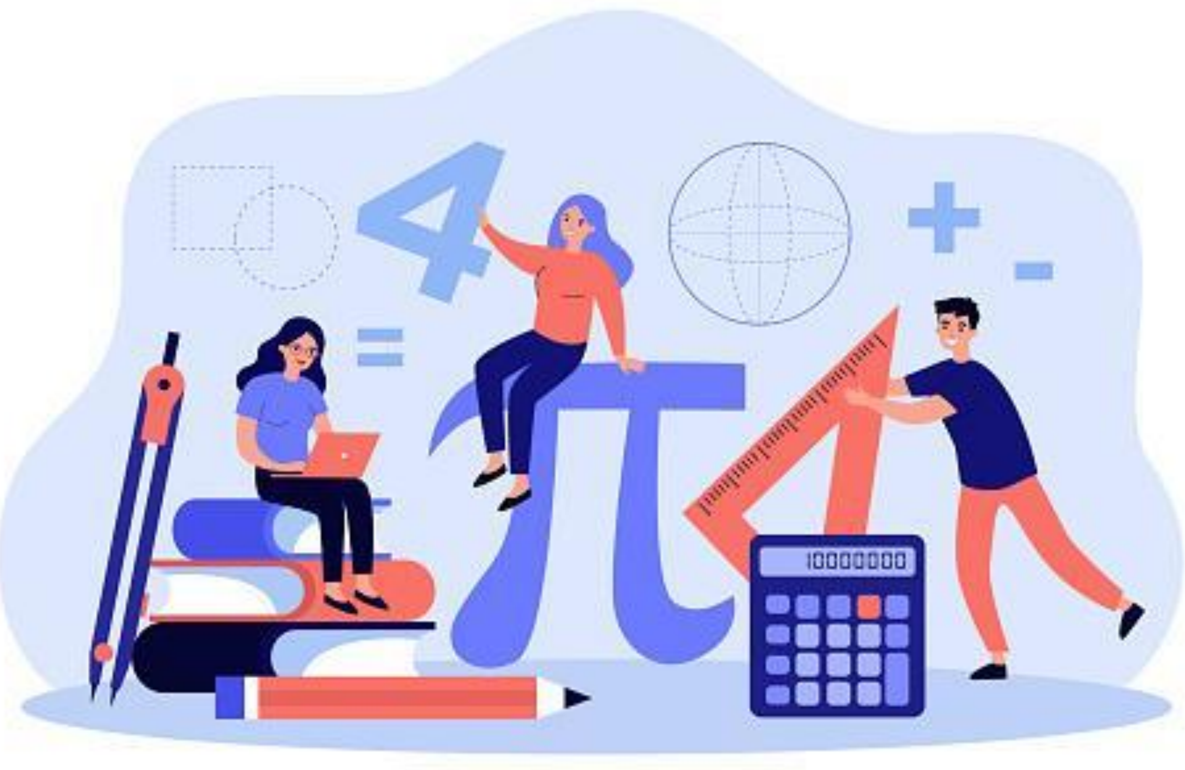
MATHEMATICS

Chapter 2: Polynomials



Polynomials

1. A **polynomial** p(x) in one variable x is an algebraic expression in x of the form $p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots + a_2x^2 + a_1x + a_0$, where.
   i. $a_0, a_1, a_2 \ldots. a_n$ are constants
   ii. x is a variable
   iii. $a_0, a_1, a_2, \ldots.. a_n$ are respectively the **coefficients** of x^i
   iv. Each of a_nx^n, a_{n-1}, x^{n-1}, $a_{n-2}x^{n-2}$, ….. a_2x^2, a_1x, a, 0 with $a_n \neq 0$, is called a **term** of a polynomial.
2. The highest power of the variable in a polynomial is called the **degree** of the polynomial.
3. A polynomial with one term is called a **monomial**.
4. A polynomial with two terms is called a **binomial**.
5. A polynomial with three terms is called a **trinomial**.
6. A polynomial with degree zero is called a **constant polynomial**. For example: 1, -3. The degree of non-zero constant polynomial is zero
7. A polynomial of degree one is called a **linear polynomial**. It is of the form ax + b. For example: x - 2, 4y + 89, 3x - z.
8. A polynomial of degree two is called a **quadratic polynomial**. It is of the form $ax^2 + bx + c$. where a, b, c are real numbers and $a \neq 0$ For example: $x^2 - 2x + 5$ etc.
9. A polynomial of degree three is called a **cubic polynomial** and has the general form $ax^3 + bx^2 + cx + d$. For example: $x^2 + 2x^2 - 2x + 5$ etc.
10. A **bi-quadratic polynomial** p(x) is a polynomial of degree four which can be reduced to quadratic polynomial in the variable $z = x^2$ by substitution.
11. The constant polynomial 0 is called the **zero polynomial**. Degree of zero polynomial is not defined.
12. The **value of a polynomial** f(x) at x = p is obtained by substituting x = p in the given polynomial and is denoted by f(p).
13. A real number ‘a’ is a **zero** or root of a polynomial p(x) if p (a) = 0.
14. The number of real zeroes of a polynomial is less than or equal to the degree of polynomial.

15. Finding a zero or root of a polynomial f(x) means solving the polynomial equation f(x) = 0.

16. A non-zero constant polynomial has no zero.

17. Every real number is a zero of a zero polynomial.

18. **Division algorithm**

If p(x) and g(x) are the two polynomials such that degree of p(x) ≥ degree of g(x) and g(x) ≠ 0, then we can find polynomials q(x) and r(x) such that:

p(x) = g(x) q(x) + r(x)

where, r(x) =0 or degree of r(x) < degree of g(x).

19. **Remainder theorem**

Let p(x) be any polynomial of degree greater than or equal to one and let a be any real number. If p(x) is divided by the linear polynomial (x – a), then remainder is p(a).

If the polynomial p(x) is divided by (x + a), the remainder is given by the value of p (-a).

If p(x) is divided by ax + b = 0; a ≠ 0, the remainder is given by

$$P\left(\frac{|-b|}{a}\right); a \neq 0$$

If p (x) is divided by ax - b = 0, a ≠ 0, the remainder is given by

$$P\left(\frac{b}{a}\right); a \neq 0$$

20. **Factor theorem**

Let p(x) is a polynomial of degree n ≥ 1 and a is any real number such that p(a) = 0, then (x - a) is a factor of p(x).

21. **Converse of factor theorem**

Let p(x) is a polynomial of degree n ≥ 1 and a is any real number. If (x - a) is a factor of p(x), then p(a) = 0.

i. (x + a) is a factor of a polynomial p(x) iff p(-a) = 0.

ii. (ax - b) is a factor of a polynomial p(x) iff p(b/a) = 0.

iii. (ax + b) is a factor of a polynomial p(x) iff p(-b/a) = 0.

iv. (x - a)(x - b) is a factor of a polynomial p(x) iff p(a) = 0 and p(b) = 0.

22. For applying factor theorem, the divisor should be either a linear polynomial of the form (ax + b) or it should be reducible to a linear polynomial.

23. A quadratic polynomial $ax^2 + bx + c$ is **factorised by splitting the middle term** by writing b as ps + qr such that (ps) (qr) = ac.

    Then, $ax^2 + bx + c = (px + q)(rx + s)$

24. An **algebraic identity** is an algebraic equation which is true for all values of the variables occurring in it.

25. Some useful **quadratic identities**:

    i. $(x + y)^2 = x^2 + 2xy + y^2$

    ii. $(x - y)^2 = x^2 - 2xy + y^2$

    iii. $(x - y)(x - y) = x^2 - y^2$

    iv. $(x + a)(x + b) = x^2 + (a + b)x + ab$

    v. $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

    Here x, y, z are variables and a, b are constants.

26. Some useful **cubic identities**:

    i. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

    ii. $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

    iii. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

    iv. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

    v. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

    vi. if $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

    Here, x, y and z are variables.

Polynomial

Polynomials are expressions with one or more terms with a non-zero coefficient. A polynomial can have more than one term. In the polynomial, each expression in it is called a term. Suppose $x^2 + 5x + 2$ is polynomial, then the expressions x^2, 5x, and 2 are the terms of the polynomial. Each term of the polynomial has a coefficient. For example, if 2x + 1 is the polynomial, then the coefficient of x is 2.

The real numbers can also be expressed as polynomials. Like 3, 6, 7, are also polynomials without any variables. These are called constant polynomials. The constant polynomial 0 is called zero polynomial. The exponent of the polynomial should be a whole number. For example, $x^{-2} + 5x + 2$, cannot be considered as a polynomial, since the exponent of x is -2, which is not a whole number.

The highest power of the polynomial is called the degree of the polynomial. For example, in $x^3 + y^3 + 3xy (x + y)$, the degree of the polynomial is 3. For a non-zero constant polynomial, the degree is zero. Apart from these, there are other types of polynomials such as:

Polynomials in One Variable

The formulas with only one variable are known as polynomials in one variable. A polynomial is a mathematical statement made up of variables and coefficients that involves the operations of addition, subtraction, multiplication, and exponentiation.

Below are Some Instances of Polynomials in One Variable:

$x^2 + 3x - 2$

$3y^3 + 2y^2 - y + 1$

$m^4 - 5m^2 + 8m - 3$

Coefficient of Polynomials.

A coefficient is a number or quantity that is associated with a variable. It's generally an integer multiplied by the variable immediately adjacent to it.

For example, in the expression 3x,3 is the coefficient but in the expression $x^2 + 3$, 1 is the coefficient of x^2.

Terms of Polynomial.

Polynomial terms are the portions of the equation that are usually separated by "+" or "-" marks. As a result, each term in a polynomial equation is a component of the polynomial. The number of terms in a polynomial like $2^2 + 5 + 4$ is 3.

Types of Polynomials:

Types of Polynomials.	Meaning	Example
Zero or constant polynomial	A constant polynomial has its coefficients equal to 0. Whereas a zero polynomial is the additive identity of the additive groups of polynomials such as f (x) = 0. In a constant polynomial, the degree is 0 whereas in a zero polynomial, the degree is undefined or written as -1.	3 or $3x^0$
Linear polynomial	Linear polynomials are polynomials having a degree of 1 as the degree of the polynomial. The greatest exponent of the variable(s) in linear polynomials is 1.	$x + y4$ $5m + 7n$ $2p$
Quadratic polynomial	Quadratic polynomials are polynomials having a degree of 2 as the degree of the polynomial.	$8x^2 + 7y - 9$ $m^2 + mn - 6$
Cubic polynomial	Cubic polynomials are polynomials having a degree of 3 as the degree of the polynomial.	$3x^3$ $p^3 + pq + 7$

Degree of Polynomial

The largest exponential power in a polynomial equation is called its degree. Only variables are taken into account when determining the degree of any polynomial; coefficients are ignored.

$4x^5 + 2x^3 – 20$

In the above polynomial degree will be 5.

Zeros of Polynomials

The polynomial zeros are the x values that fulfil the equation y = f(x). The zeros of the polynomial are the values of x for which the y value is equal to zero, and f(x) is a function of x. The degree of the equation y = f(x), determines the number of zeros in a polynomial.

Factorization of Polynomials

You know that any polynomial of the form p(a) can also be written as p(a) = g(a) × h(a) + R(a)

Dividend = Quotient × Divisor + Remainder

If the remainder is zero, then p(a) = g(a) x h(a). That is, the polynomial p(a) is a product of two other polynomials g (a) and h(a). For example, $3a + 6a^2 = 3a \times (1 + 2a)$.

A polynomial may be expressed in more than one way as the product of two or more polynomials.

Study the polynomial $3a + 6a^2 = 3a \times (1 + 2a)$.

This can also be factorised as $3a + 6a^2 = 6a \times \left(\frac{1}{2} + a\right)$.

Methods of Factorizing Polynomials

A polynomial can be factorised in a number of ways.

- Factorization, which is done by dividing the expression by the HCF of the words in the provided expression.
- Factorization by grouping the terms of the expression.
- Factorization using identities.

Factorization is achieved by dividing the expression by the HCF of the given expression's terms.

The biggest monomial in a polynomial is the HCF, which is a factor of each term in the polynomial. We can factorise a polynomial by determining the expression's Highest Common Factor (HCF) and then dividing each term by its HCF. The factors of the above equation are HCF and the quotient achieved.

Steps for Factorization

- Determine the HCF of the supplied expression's terms.
- Find the quotient by dividing each term of the provided equation by the HCF.
- As a product of HCF and quotient, write the given expression.

Factorization by Grouping the Expression's Terms

We come encounter polynomials in a variety of circumstances, and they may or may not contain common factors among their components. In such instances, we arrange the expression's terms so that common factors exist among the terms of the resulting groups.

Steps for Factorization by Grouping

- If required, rearrange the terms.
- Assemble the provided phrase into groups, each with its own common component.
- Determine each group's HCF.
- Find out what the other component is.
- Convert the phrase to a product of the common and additional factors.

Factorization Using Identities

To Locate the Products, Recall the Following Identities:

1.$(a+b)^2=a^2+2ab+b^2$

2. $(a-b)^2=a^2-2ab+b^2$

3. $(a+b)(a-b)=a^2-b^2$

4. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

5. $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

6.$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Observe that the LHS in the identities are all factors and the RHS are their products. Thus, we can write the factors as follows:

Factors of $a^2-2ab+b^2$ are $(a-b)$ and $(a-b)$ Factors of $a^2+2ab+b^2$ are $(a+b)$ and $(a+b)$ Factors of a^2-b^2 are $(a+b)$ and $(a-b)$ Factors of $a^3+3a^2b+3ab^2+b^3$ are $(a+b),(a+b)$ and $(a + b)

Factors of $a^3-3a^2b+3ab^2-b^3$ are $(a-b),(a-b)$ and $(a-b)$ Factors of $a^2+b^2+c^2+2ab+2bc+2ca \text{ are} (a+b+c)$ and $(a+b+c)$

We may deduce from the preceding identities that a given statement in the form of an identity can be expressed in terms of its components.

Steps for Factorization Using Identities

Recognize the correct persona.

In the form of the identity, rewrite the provided statement.

Using the identity, write the factors of the given equation.

$$a^3 \pm b^3 \pm 3ab(a \pm b) = (a \pm b)^3$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$x^3 \pm y^3 = (x \pm y)\left(x^2 \pm xy + y^2\right)$$

Factorization of Trinomials of the Form $x^2 + bx + c$

Trinomials are expressions with three terms. For example, $x^2 + 14x + 49$ is a trinomial. All trinomials cannot be factorised using a single approach. We must investigate the pattern in trinomials and select the best approach for factorising the given trinomial.

Factorizing a Trinomial by Splitting the Middle Term

The product of two binomials of the type (x + a) and (x + b) is $(x + a) \times (x + b)\ x^2 + x(a + b) + ab$ [a trinomial]

CHAPTER : 2 POLYNOMIALS

Polynomials

Polynomial

An algebraic expression of the form:
$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x^1 + a_0 x^0$

Polynomial in one variable:
$ax^3 - bx^2 - cx + d$

Algebraic Identities

$(x+y)^2$	$x^2 + 2xy + y^2$
$(x-y)^2$	$x^2 - 2xy + y^2$
$x^2 - y^2$	$(x - y)(x + y)$
$(x+a)(x+b)$	$x^2 + (a + b)x + ab$
$(x + y + z)^2$	$x^2+y^2+z^2+2xy+2yz+2zx$ $= x^2+y^2+z^2+2(xy+yz+zx)$
$(x + y)^3$	$x^3 + y^3 + 3xy(x + y)$
$(x - y)^3$	$x^3 - y^3 - 3xy(x - y)$
$x^3+y^3+z^3-3xyz$	$(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$
If $x + y + z = 0$	(i) $x^3+y^3+z^3 = 3xyz$ (ii) $\frac{x^2}{yz} + \frac{y^2}{xz} + \frac{z^2}{xy} = 3$
$x^3 + y^3$	$(x+y)(x^2-xy+y^2)$
$x^3 - y^3$	$(x-y)(x^2+xy+y^2)$

Types

Terms

Polynomial	Example
Constant (or independent)	4, − 7/5
Zero	Degree not defined (constant polynomial 0)
Monomial	$4x$
Binomial	$2x + 3$
Trinomial	$3x^2 + 7x + 2$

Degree

Polynomial	Example	Degree
Linear	$3x + 2$	1
Quadratic	$2x^2 + 3x + 1$	2
Cubic	$7y^3 + 6y^2 + 2y + 2$	3

Theorems

Factor Theorem

if $p(x)$ is a polynomial of degree $n \geq 1$, a: any real number

(i) if $(x-a)$ is a factor of $p(x)$, then $p(a) = 0$

(ii) if $p(a) = 0$, then $(x-a)$ is a factor of $p(x)$.

Remainder Theorem

if $p(x)$ polynomial of degree $n>1$, is divided by $x-a$, $p(a)$ is the remainder

Divident = (Divisor × Quotient) + Remainder

Zeroes of polynomial

Number that satisfies the equation

Example

$P(x) = 2x + 1$
find zeroes of the polynomial

$P(x) = 0$
$2x + 1 = 0$
$x = -1/2$

$-1/2$ is the zeros of the polynomial

Important Questions

Multiple Choice Questions-

Question. 1 x^2-2x +1 is a polynomial in:

a. One Variable

b. Two Variables

c. Three variable

d. None of the above

Question. 2 The coefficient of x^2 in $3x^3+2x^2-x+1$ is:

a. 1

b. 2

c. 3

d. -1

Question. 3 A binomial of degree 20 in the following is:

a. 20x + 1

b. x/20 + 1

c. x20 +1

d. x2+20

Question. 4 The degree of $4x^3-12x^2+3x+9$ is

a. 0

b. 1

c. 2

d. 3

Question. 5 $x^2 – x$ is _________ polynomial.

a. Linear

b. Quadratic

c. Cubic

d. None of the above

Question. 6 $x – x^3$ is a _________ polynomial.

a. Linear

b. Quadratic

c. Cubic

d. None of the above

Question. 7 1 + 3x is a __________ polynomial.

a. Linear

b. Quadratic

c. Cubic

d. None of the above

Question. 8 The value of $f(x) = 5x-4x^2+3$ when x = -1, is:

a. 3

b. -12

c. -6

d. 6

Question. 9 The value of $p(t) = 2+t+2t^2-t^3$ when t=0 is

a. 2

b. 1

c. 4

d. 0

Question. 10 The zero of the polynomial f(x) = 2x+7 is

a. 2/7

b. -2/7

c. 7/2

d. -7/2

Very Short:

1. Factorise: $125x^3 - 64y^3$
2. Find the value of $(x + y)^2 + (x - y)^2$.
3. If $p(x)= x^2 - 2\sqrt{2}x+1$, then find the value of $p(2\sqrt{2})$
4. Find the value of m, if x + 4 is a factor of the polynomial $x^2 + 3x + m$.
5. Find the remainder when $x^3+ x^2 + x + 1$ is divided by $x - \frac{1}{2}$ using remainder theorem.
6. Find the common factor in the quadratic polynomials $x^2 + 8x + 15$ and $x^2 + 3x - 10$.

Short Questions:

1. Expand:

   (i) $(y - \sqrt{3})^2$

(ii) $(x - 2y - 3z)^2$

2. If, $x + = \frac{1}{x} . = 7$

3. then find the value of $x^3 + \frac{1}{x^3}$

4. Show that p – 1 is a factor of $p^{10} + p^8 + p^6 - p^4 - p^2 - 1$.

5. If 3x + 2y = 12 and xy = 6, find the value of $27x^3 + 8y^3$

6. Factorise: $4x^2 + 9y^2 + 16z^22 + 12xy - 24\ yz - 16xz$.

7. Factorise: $1 - 2ab - (a^2 + b^2)$.

8. Factories:

$$27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}.$$

Long Questions:

1. Prove that $(a + b + c)^3 - a^3 - b^3 - c^3 = 3(a + b) (b + c) (c + a)$.
2. Factorise: $(m + 2n)^2 x^2 - 22x (m + 2n) + 72$.
3. If x – 3 is a factor of $x^2 - 6x + 12$, then find the value of k. Also, find the other factor of the – polynomial for this value of k.
4. Find a and b so that the polynomial $x^3 - 10x^2 + ax + b$ is exactly divisible by the polynomials (x – 1) and (x – 2).
5. Factorise: $x^2 - 6x^2 + 11x - 6$.

Assertion and Reason Questions:

1. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: If $f(x) = 3x^7 - 4x^6 + x + 9$ is a polynomial, then its degree is 7.

Reason: Aromatic aldehydes are almost as reactive as formaldehyde.

2. In these questions, a statement of assertion followed by a statement of reason is given. Choose the correct answer out of the following choices.

a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.

c) Assertion is correct statement but reason is wrong statement.

d) Assertion is wrong statement but reason is correct statement.

Assertion: The expression $3x^4 – 4x^{3/2} + x^2 = 2$ is not a polynomial because the term $– 4x^{3/2}$

contains a rational power of x.

Reason: The highest exponent in various terms of an algebraic expression in one variable is called its degree.

Answer Key:

MCQ:

1. (a) One Variable
2. (b) 2
3. (c) $x^{20} +1$
4. (d) 3
5. (b) Quadratic
6. (c) Cubic
7. (a) Linear
8. (c) -6
9. (a) 2
10. (d) -7/2

Very Short Answer:

1. $125x^3– 6443 = (5x)^3 – (4y)^3$

   By using $a3 – b3 = (a – b) (a^2 + a^b + b^2)$, we obtain

   $125x^3– 64y3 = (5x – 4y) (25x2 + 20xy + 16y^2)$

2. $(x + y)^2 + (x – y)^2 = x^2 + y^2 + 2xy + x^2 + y^2 – 2xy$

$= 2x^2 + 2y^2 = 21x^2 + y^2)$

3. Put x = 2√2 in p(x), we obtain

$p(2\sqrt{2}) = (2\sqrt{2})^2 – 2\sqrt{2}(2\sqrt{2}) + 1 = (2\sqrt{2})^2 – (2\sqrt{2})^2 + 1 = 1$

4. Let $p(x) = x^2 + 3x + m$

   Since (x + 4) or (x – (-4)} is a factor of p(x).

   ∴ p (-4) = 0

$\Rightarrow (-4)^2 + 3(-4) + m = 0$

$\Rightarrow 16 - 12 + m = 0$

$\Rightarrow m = -4$

5. Let p(x) = x3+ x2 + x + 1 and q(x) = $x - \frac{1}{2}$

   Here, p(x) is divided by q(x)

   ∴ By using remainder theorem, we have

$$\text{Remainder} = p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 + \frac{1}{2} + 1$$

$$= \frac{1}{8} + \frac{1}{4} + \frac{1}{2} + 1 = \frac{1+2+4+8}{8} = \frac{15}{8}.$$

6. $x^2 + 8x + 15 = x^2 + 5x + 3x + 15 = (x + 3)(x + 5)$

   $x^2 + 3x - 10 = x^2 + 5x - 2x - 10 = (x - 2)(x + 5)$

   Clearly, the common factor is x + 5.

Short Answer:

Ans: 1. $(y - \sqrt{3})^2$ = y2 -2 × y × √3 + $(\sqrt{3})^2 = y^2 - 2\sqrt{3}\,y + 3$ $(x - 2y - 3z)^2$

$= x^2 + 1 - 2y)^2 + (-3z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-3z) + 2 \times (-3z) \times x$

$= x^2 + 4y^2 + 9z^2 - 4xy + 12yz - 6zx$

Ans: 2. We have $x + \frac{1}{x} = 7$

Cubing both sides, we have

$$\left(x+\frac{1}{x}\right)^3 = 7^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x+\frac{1}{x}\right) = 343$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 7 = 343$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 343 - 21 = 322$$

Ans: 3. Let $f(p) = p^{10} + p^8 + p^6 - p^4 - p^2 - 1$

Put p = 1, we obtain

$f(1) = 1^{10} + 1^8 + 1^6 - 1^4 - 1^2 - 1$

$= 1 + 1 + 1 - 1 - 1 - 1 = 0$

Hence, p – 1 is a factor of $p^{10} + p^8 + p^6 - p^4 - p^2 - 1$

Ans: 4. We have $3x + 2y = 12$

On cubing both sides, we have

$\Rightarrow (3x + 2y)^3 = 12^3$

$\Rightarrow (3x)^3 + (2y)^3 + 3 \times 3x \times 2y(3x + 2y) = \sqrt{728}$

$\Rightarrow 27x^3 + 8y^3 + 18xy(3x + 2y) = \sqrt{728}$

$\Rightarrow 27x^3 + 8y^3 + 18 \times 6 \times 12 = \sqrt{728}$

$\Rightarrow 27x^3 + 8y^3 + 1296 = \sqrt{728}$

$\Rightarrow 27x^3 + 8y^3 = \sqrt{728} - 1296$

$\Rightarrow 27x^3 + 8y^3 = 432$

Ans: 5. $4x^2 + 9y^2 + 16z22 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(= 42) + 2(- 42)(2x)$

By using $a^2 + b^2 + 2ab + 2bc + 2ca = (a + b + c)^2$, we obtain

$= (2x + 3y - 4z)^2 = (2x + 3y - 4z)\ (2x + 3y - 4z)$

Ans: 6. $1 - 2ab - (a^2 + b^2) = 1 - (a^2 + b^2 + 2ab)$

$= 1^2 - (a + b)^2$

$= (1 + a + b)\ (1 - a - b)$

$[\because x^2 - y^2 = (x + y)\ (x - y)]$

Ans: 7.

$$27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2} = (3a)^3 + \frac{1}{(4b)^3} + 3.(3a).\left(\frac{1}{4b}\right)\left(3a + \frac{1}{4b}\right)$$

By using $x^3 + y^3 + 3xy\ (x + y) = (x + y)^3$, we have

$$= \left(3a + \frac{1}{4b}\right)^3.$$

Long Answer:

Ans: 1. L.H.S. $= (a + b + c)^3 - a^3 - b^3 - c^3$

$= \{(a + b + c)^3 - 3\} - \{b^3 + c^3\}$

$= (a + b + c - a)\ \{(a + b + c)^2 + a^2 + a(a + b + c)\} - (b + c)\ (b^2 + c^2 - bc)$

$= (b + c)\ \{a^2 + b^2 + 2 + 2ab + 2bc + 2ca + a^2 + a^2 + ab + ac - b^2 - a^2 + bc)$

$= (b + c)\ (3a2 + 3ab + 3bc + 3ca\}$

$= 3(b + c)\ \{a^2 + ab + bc + ca\}$

$= 31b + c)\ \{\{a^2 + ca) + (ab + bc)\}$

$= 3(b + c)\ \{a(a + c) + b(a + c)\}$

$= 3(b + c)(a + c)(a + b)$

$= 3(a + b)(b + c)(c + a) =$ R.H.S.

Ans: 2. Let $m + 2n = a$

$\therefore (m + 2n)^2 x^2 - 22x(m + 2n) + 72 = a^2x^2 - 22ax + 72$

$= a^2x^2 - 18ax - 4ax + 72$

$= ax(ax - 18) - 4(ax - 18)$

$= (ax - 4)(ax - 18)$

$= \{(m + 2n)x - 4)\}\{(m + 2n)x - 18)\}$

$= (mx + 2nx - 4)(mx + 2nx - 18).$

Ans: 3. Here, $x - 3$ is a factor of $x^2 - kx + 12$

$\therefore$ By factor theorem, putting $x = 3$, we have remainder 0.

$\Rightarrow (3)^2 - k(3) + 12 = 0$

$\Rightarrow 9 - 3k + 12 = 0$

$\Rightarrow 3k = 21$

$\Rightarrow k = 7$

Now, $x^2 - 7x + 12 = x^2 - 3x - 4x + 12$

$= x(x - 3) - 4(x - 3)$

$= (x - 3)(x - 4)$

Hence, the value of k is 7 and other factor is $x - 4$.

Ans: 4. Let $p(x) = x^3 - 10x^2 + ax + b$

Since p(x) is exactly divisible by the polynomials $(x - 1)$ and $(x - 2)$.

$\therefore$ By putting $x = 1$, we obtain

$(1)^3 - 10(1)^2 + a(1) + b = 0$

$\Rightarrow a + b = 9$

And by putting $x = 2$, we obtain

$(2)^3 - 10(2)^2 + a(2) + b = 0$

$8 - 40 + 2a + b = 0$

$\Rightarrow 2a + b = 32$

Subtracting (i) from (ii), we have

$a = 23$

From (i), we have $23 + b = 9 = b = -14$

Hence, the values of a and b are $a = 23$ and $b = -14$

Ans: 5. Let $p(x) = x^2 - 6x^2 + 11x - 6$

Here, constant term of p(x) is -6 and factors of -6 are ± 1, ± 2, ± 3 and ± 6

By putting x = 1, we have

$p(1) = (1)^3 - 6(1)^2 + 11(1) - 6 = 1 - 6 + 11 - 6 = 0$

∴ (x – 1) is a factor of p(x)

By putting x = 2, we have

$p(2) = (2)^3 - 6(2)^2 + 11(2) - 6 = 8 - 24 + 22 - 6 = 0$

∴ (x – 2) is a factor of p(x)

By putting x = 3, we have

$p(3) = (3)^3 - 6(3)^2 + 11(3) - 6 = 27 - 54 + 33 - 6 = 0$

∴ (x – 3) is a factor of p(x) Since p(x) is a polynomial of degree 3, so it cannot have more than three linear factors.

∴ $x^3 - 6x^2 + 11x - 6 = k(x - 1)(x - 2)(x - 3)$

By putting x = 0, we obtain

$0 - 0 + 0 - 6 = k(-1)(-2)(3)$

$-6 = -6k$

$k = 1$

Hence, $x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$.

Assertion and Reason Answers:

1. a) Assertion and reason both are correct statements and reason is correct explanation for assertion.

2. b) Assertion and reason both are correct statements but reason is not correct explanation for assertion.